CASUALTY ACTUARIAL SOCIETY
AND THE
CANADIAN INSTITUTE OF ACTUARIES

Exam 3, Segment 3L

Life Contingencies and Statistics

May 9, 2008

2.5 HOURS

INSTRUCTIONS TO CANDIDATES

1. This 50 point examination consists of 25 multiple choice questions worth 2 points each.

2. To answer the multiple choice questions, use the short-answer card provided and a number 2 or HB pencil only.

   - Fill in that it is Spring 2008 and that the exam number is 3.
   - Darken the spaces corresponding to your Candidate ID number. Four rows are available. If your Candidate ID number is fewer than 4 digits, include leading zeros. For example, if your Candidate ID number is 987, consider that your Candidate ID number is 0987, enter a zero on the first row, 9 on the second row, 8 on the third row, and 7 on the fourth [last] row. Write in your Candidate ID number next to the place where you darken the spaces for your Candidate ID number. Your name, or any other identifying mark, must not appear on the short-answer card.
   - Mark your short-answer card during the examination period. No additional time will be allowed for this after the exam has ended. Make your marks dark and fill in the spaces completely.
   - For each of the multiple choice questions, select the one best answer and fill in the corresponding letter. One quarter of the point value of the question will be subtracted for each incorrect answer. No points will be added or subtracted for responses left blank.

3. Do all problems until you reach the last page of the examination where "END OF EXAMINATION" is marked.

4. Prior to the start of the exam you will have a **ten-minute reading period** in which you can silently read the questions and check the exam booklet for missing or defective pages. Writing will NOT be permitted during this time and you will not be permitted to hold pens or pencils. You will also not be allowed to use calculators. The supervisor has additional exams for those candidates who have defective exam booklets.

   - Verify that you have a copy of "Tables for CAS Exam 3" included in your exam packet.

CONTINUE TO NEXT PAGE OF INSTRUCTIONS

5. Your Examination Envelope is pre-labeled with your Candidate ID number, name, exam number, and test center. Do not remove this label. Keep a record of your Candidate ID number for future inquiries regarding this exam.

6. Candidates must remain in the examination center until the examination has concluded. The examination starts after the reading period is complete. You may leave the examination room to use the restroom with permission from the supervisor.

7. At the end of the examination, place the short-answer card in the Examination Envelope. Nothing written in the examination booklet will be graded. Only the short-answer card will be graded. Also place any included reference materials in the Examination Envelope. BEFORE YOU TURN THE EXAMINATION ENVELOPE IN TO THE SUPERVISOR, BE SURE TO SIGN IT IN THE SPACE PROVIDED ABOVE THE CUT-OUT WINDOW.

8. If you have brought a self-addressed, stamped envelope, you may put the examination booklet and scrap paper inside and submit it separately to the supervisor. It will be mailed to you. Do not put the self-addressed stamped envelope inside the Examination Envelope.

   If you do not have a self-addressed, stamped envelope, please place the examination booklet in the Examination Envelope and seal the envelope. You may not take it with you. Do not put scrap paper in the Examination Envelope. The supervisor will collect your scrap paper.

   Candidates may obtain a copy of the examination from the CAS Web Site.

   All extra answer sheets, scrap paper, etc. must be returned to the supervisor for disposal.

9. Candidates must not give or receive assistance of any kind during the examination. Any cheating, any attempt to cheat, assisting others to cheat, or participating therein, or other improper conduct will result in the Casualty Actuarial Society and the Canadian Institute of Actuaries disqualifying the candidate's paper, and such other disciplinary action as may be deemed appropriate within the guidelines of the CAS Policy on Examination Discipline.

10. The exam survey is available on the CAS Web Site in the "Admissions/Exams" section. Please submit your survey by June 2, 2008.

END OF INSTRUCTIONS

1. Agent ABC is informed by insurer XYZ that if ABC succeeds in selling XYZ's insurance to at least 56 customers before December 31, ABC will receive a bonus.

- ABC will make a sales pitch to 500 customers before December 31.
- ABC's probability of success in selling XYZ's policy to each customer is 10%.
- The central limit theorem asserts that for any distribution with a sample size n from any distribution having finite variance , the random variable converges in distribution to a random variable having a standard normal distribution

Using the central limit theorem, estimate the probability that ABC will succeed in earning the bonus.

A. Less than 18%
B. At least 18% but less than 22%
C. At least 22% but less than 26%
D. At least 26% but less than 30%
E. At least 30%

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2. X is a Pareto random variable with the true parameter $\theta = 5{,}000$. A computer simulation of n claims will produce the following estimator of θ:

$$\hat{\theta}_n = 5{,}000 * n/(n+1), \text{ for n} = 1,2,\ldots$$

Which of the following statements are true about the estimator $\hat{\theta}_n$?

I. $\hat{\theta}_n$ is an unbiased estimator of θ.

II. $\hat{\theta}_n$ is a consistent estimator of θ.

III. The mean square error of $\hat{\theta}_{10}$ is more than 200,000.

A. None are true
B. I and II only
C. I and III only
D. II and III only
E. I, II, and III

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3. You are given the following:

- A random sample of claim amounts:

  8,000 10,000 12,000 15,000

- Claim amounts follow an inverse exponential distribution, with parameter θ

Calculate the maximum likelihood estimator for θ.

A. Less than 9,000
B. At least 9,000, but less than 10,000
C. At least 10,000, but less than 11,000
D. At least 11,000, but less than 12,000
E. At least 12,000

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4. One hundred insureds are assigned to one of three classes based on prior policy period claim experience, as follows:

Class	Number of Claims in Prior Policy Period	Number of Insureds in Class
A	No Claims	74
B	One Claim	16
C	Two or More Claims	10

Calculate the value of the chi square statistic which results from testing the hypothesis that claim frequency follows a Poisson distribution with mean 0.4.

A. Less than 2
B. At least 2, but less than 4
C. At least 4, but less than 6
D. At least 6, but less than 8
E. At least 8

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5. Let X be a random variable. X is normally distributed with $\sigma = 1.5$ and either $\mu=1$ or $\mu=5$.

Consider the following hypotheses:

H_0: X is normally distributed with $\mu=1$ and $\sigma=1.5$.
H_1: X is normally distributed with $\mu=5$ and $\sigma=1.5$.

You perform hypothesis testing by observing one value of X and rejecting H_0 if this value exceeds k.

If the probability of a Type I error is 2.5%, calculate the probability of a Type II error.

A. Less than 10%
B. At least 10%, but less than 20%
C. At least 20%, but less than 30%
D. At least 30%, but less than 40%
E. At least 40%

CONTINUED ON NEXT PAGE

6. You are given a random sample from a normal loss distribution:

- The sample mean is 42,000
- The sample standard deviation is 8,000
- There are 25 loss observations in the sample

Using a two-sided test with $H_0 = 45{,}000$ versus $H_1 \neq 45{,}000$, at which value of α would you reject the null hypothesis?

A. Reject at $\alpha = .01$
B. Do not reject at $\alpha = .01$, but reject at $\alpha = .02$
C. Do not reject at $\alpha = .02$, but reject at $\alpha = .05$
D. Do not reject at $\alpha = .05$, but reject at $\alpha = .1$
E. Do not reject at $\alpha = .1$

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7. You are given:

- A random sample of three claims
- The claim amount distribution is uniform over $50 < x < 700$

Calculate the probability that no more than two claims will exceed 300.

A. Less than 40%
B. At least 40% but less than 50%
C. At least 50% but less than 60%
D. At least 60% but less than 70%
E. At least 70%

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8. Claim severity follows an exponential distribution with mean 1000. The probability density function of the order statistic Y_k with a sample size n is

$$\frac{n!}{(k-1)!(n-k)!}[F(y)]^{k-1}[1-F(y)]^{n-k}f(y).$$

If two claims are sampled randomly, calculate the expected value of the larger claim.

A. Less than 1200
B. At least 1200, but less than 1400
C. At least 1400, but less than 1600
D. At least 1600, but less than 1800
E. At least 1800

CONTINUED ON NEXT PAGE

9. You are modeling the price of gold, Y, as a function of the price of silver, X, using a linear regression. You are given the following data points and summary statistics:

Price of Silver (X)	Price of Gold (Y)
\$4.67	\$343.80
\$5.97	\$416.25
\$6.39	\$427.75
\$9.04	\$530.00
\$13.01	\$639.75

$$\sum_{i=1}^{5} X_i = 39.08 \qquad \sum_{i=1}^{5} Y_i = 2{,}357.55 \qquad \hat{\beta} = 34.5$$

Calculate the residual at (6.39,427.75).

A. Less than –15
B. At least –15, but less than –5
C. At least –5, but less than 5
D. At least 5, but less than 15
E. At least 15

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10. Car accidents follow a Poisson process, as described below:

- On Monday and Friday, the expected number of accidents per day is 3.
- On Tuesday, Wednesday, and Thursday, the expected number of accidents per day is 4.
- On Saturday and Sunday, the expected number of accidents per day is 1.

Calculate the probability that exactly 18 accidents occur in a week.

A. Less than .06
B. At least .06 but less than .07
C. At least .07 but less than .08
D. At least .08 but less than .09
E. At least .09

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11. A customer service call center operates from 9:00 AM to 5:00 PM. The number of calls received by the call center follows a Poisson process whose rate function varies according to the time of day, as follows:

Time of Day	Call Rate (per hour)
9:00 AM to 12:00 PM	30
12:00 PM to 1:00 PM	10
1:00 PM to 3:00 PM	25
3:00 PM to 5:00 PM	30

Using a normal approximation, what is the probability that the number of calls received from 9:00AM to 1:00PM exceeds the number of calls received from 1:00PM to 5:00PM?

A. Less than 10%
B. At least 10%, but less than 20%
C. At least 20%, but less than 30%
D. At least 30%, but less than 40%
E. At least 40%

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12. You are given the following information:

- Hurricane occurrences in Texas follow a Poisson process with $\lambda = 2$ per year.
- An insurance company sells 1,000,000 homeowners policies in Texas.
- The insurance company's losses per hurricane follow an exponential distribution with a mean of \$10 million.
- The "risk load" is calculated as 10% of the sum of the expected losses per year and the standard deviation per year.

Calculate the risk load per policy.

A. Less than 5 dollars
B. At least 5 dollars, but less than 10 dollars
C. At least 10 dollars, but less than 15 dollars
D. At least 15 dollars, but less than 20 dollars
E. At least 20 dollars

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13. The future lifetime of (x) is described by the following survival function:

- $s(x)=\left(\frac{\omega-x}{\omega}\right)^{\alpha}$
- $\omega=40$
- $\alpha=1.1$

Find $\mathring{e}_{30}$, the expected future lifetime for (30)

A. Less than 5.00
B. At least 5.00, but less than 5.25
C. At least 5.25, but less than 5.50
D. At least 5.50, but less than 5.75
E. At least 5.75

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14. Use the Illustrative Life Table with the following adjustments:

- $q_x^* = 4.0 \cdot q_x$ for $67 \le x \le 68$
- $q_{69}^* = q_{66}$

Calculate ${}_{2|2}q_{66}$.

A. Less than 0.1150
B. At least 0.1150 but less than 0.1175
C. At least 0.1175 but less than 0.1200
D. At least 0.1200 but less than 0.1225
E. At least 0.1225

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15. You are given the following survival function:

$$s(x) = e^{-5x^7}$$

Calculate $\mu(x)$, the force of mortality.

A. $5x^7$

B. $35x^6$

C. $35x^6\, e^{-5x^7}$

D. $5x^7 \ln(35x^6)$

E. $\dfrac{35x^6 e^{-5x^7}}{1-e^{-5x^7}}$

16. You are given the following life table:

x	ℓ_x
45	1,000
46	900
47	740

Use the Uniform Distribution of Deaths (UDD) assumption for intermediate ages.

Calculate: ${}_{0.5|0.5}q_{45.25}$

A. Less than 0.055
B. At least 0.055, but less than 0.060
C. At least 0.060, but less than 0.065
D. At least 0.065, but less than 0.070
E. At least 0.070

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17. For two independent lives *(x)* and *(y)*, you are given:

- T_x and T_y follow Weibull distributions
- Hazard rate functions $\lambda_x = kx^4$ and $\lambda_y = ky^4$
- Joint survival probability $p_{x:y}$

Calculate *k* such that the joint survival probability $p_{2:2} = 0.5$

A. Less than 0.010
B. At least 0.010, but less than 0.015
C. At least 0.015, but less than 0.020
D. At least 0.020, but less than 0.025
E. At least 0.025

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18. The future lifetime variables *T(x)* and *T(y)* are independent and identically distributed with probability density function:

$$f(z)=\frac{1}{(1+z)^2}$$

Determine the joint survival function $S_{T(x)T(y)}(s,t)$.

A. $\frac{1}{(1+s)(1+t)}$

B. $1-\frac{1}{(1+s)(1+t)}$

C. $\frac{st}{(1+s)(1+t)}$

D. $1-\frac{st}{(1+s)(1+t)}$

E. $\frac{1}{(1+s)^2(1+t)^2}$

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19. You are given a multiple decrement model with the following four forces of decrement:

$$\mu_x^{(1)}(t) = \frac{t^3}{100} \qquad t \geq 0$$
$$\mu_x^{(2)}(t) = \frac{3t^2}{100} \qquad t \geq 0$$
$$\mu_x^{(3)}(t) = \frac{3t}{100} \qquad t \geq 0$$
$$\mu_x^{(4)}(t) = \frac{1}{100} \qquad t \geq 0$$

Calculate $q_x^{(\tau)}$, the probability of decrement due to any of the four causes.

A. Less than 0.025
B. At least 0.025, but less than 0.030
C. At least 0.030, but less than 0.035
D. At least 0.035, but less than 0.040
E. At least 0.040

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20. Insureds are divided into three classes for automobile insurance:

- Class 1 = Preferred
- Class 2 = Standard
- Class 3 = Substandard

At the end of each year insureds may transition from one class to another, according to the following probability matrix:

$$Q_n^{(i,j)} = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.2 & 0.6 & 0.2 \\ 0 & 0.3 & 0.7 \end{bmatrix}$$

At time = 0, an insured is in the standard class. Calculate the probability that this insured is in the standard class at time = 2.

A. Less than 0.40
B. At least 0.40, but less than 0.45
C. At least 0.45, but less than 0.50
D. At least 0.50, but less than 0.55
E. At least 0.55

CONTINUED ON NEXT PAGE

21. You are given the following about two whole life insurance policies with unit benefit, priced at their actuarial present values.

- Policy A is priced under the assumption that lives follow an exponential distribution with mean 5.
- Policy B is priced under the assumption that lives follow a uniform distribution with maximum attained age of 80.
- $i = 0.06$

For a person aged 70, calculate the absolute difference in the price of the policies.

A. Less than 0.01
B. At least 0.01, but less than 0.02
C. At least 0.02, but less than 0.03
D. At least 0.03, but less than 0.04
E. At least 0.04

CONTINUED ON NEXT PAGE

22. You are given:

- The probability density function of the future lifetime, T, for (x) is exponential
- The force of mortality, $\mu = 0.06$
- The force of interest, $\delta = 0.03$

A whole life insurance policy of unit benefit is issued to (x), payable at the moment of death. Calculate the 80th percentile of the distribution of the present value of the benefit payment.

A. Less than 0.86
B. At least 0.86, but less than 0.87
C. At least 0.87, but less than 0.88
D. At least 0.88, but less than 0.89
E. At least 0.89

CONTINUED ON NEXT PAGE

23. You are given:

- A whole life policy on a life aged 30
- The policy pays 1,000 at the end of the year of death
- Mortality follows the Illustrative Life Table
- $i = 6\%$
- Premiums satisfy the equivalence principle with the following payment schedule:

$$P = \begin{cases} x, & t=0,1,2,3,4 \\ 2x, & t=5,6,7,\ldots \end{cases}$$

Calculate x.

A. Less than 3.60
B. At least 3.60, but less than 3.65
C. At least 3.65, but less than 3.70
D. At least 3.70, but less than 3.75
E. At least 3.75

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24. For a fully discrete 5-year endowment insurance of benefit 1 on (35), you are given the following:

- $\ddot{a}_{35:\overline{5}|} = 4.2$
- d = 0.085

Calculate ${}_4\overline{V}(\overline{A}_{35:\overline{5}|})$, the benefit reserve at t = 4.

A. Less than 0.755
B. At least 0.755 but less than 0.760
C. At least 0.760 but less than 0.765
D. At least 0.765 but less than 0.770
E. At least 0.770

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25. You are given the following information about a transition probability matrix for states numbered 1 and 2.

$$Q = \begin{bmatrix} 0.75 & 0.25 \\ 0.50 & 0.50 \end{bmatrix}$$

- Transitions occur at the end of each year
- Cash flows occur at the mid-point of each year, according to the following matrix:

$$C = \begin{bmatrix} 0 \\ 30 \end{bmatrix}$$

- $i = 6\%$
- The subject starts in state 1 at time = 0

Calculate the actuarial present value of the subject's cash flows for the first 3 years.

A. Less than 14.75
B. At least 14.75, but less than 15.00
C. At least 15.00, but less than 15.25
D. At least 15.25, but less than 15.50
E. At least 15.50

END OF EXAMINATION

Exam 3L Answer Key

1. B
2. D
3. C
4. D
5. C
6. D
7. E
8. C
9. D
10. D
11. C
12. A
13. A
14. B
15. B
16. D
17. A
18. A
19. D
20. D
21. B
22. E
23. E
24. C No penalty applied for answers other than “C”
25. B