Practice Exam 1

1. For American put options on a stock with identical expiry dates, you are given the following prices:

Strike price	Put premium
30	2.40
35	6.40

For an American put option on the same stock with the same expiry date and strike price 38, which of the following statements is correct?

(A) The lowest possible price for the option is 8.80.

(B) The highest possible price for the option is 8.80.

(C) The lowest possible price for the option is 9.20.

(D) The highest possible price for the option is 9.20.

(E) The lowest possible price for the option is 9.40.

2. A company has 100 shares of ABC stock. The current price of ABC stock is 30. ABC stock pays no dividends.

The company would like to hedge its exposure to drops in the stock price by buying European put options expiring in 6 months with exercise price 28.

European call options on the same stock expiring in 6 months with exercise price 28 are available for 4.10.

The continuously compounded risk-free rate is 5%.

Determine the cost of the hedge.

(A) 73 (B) 85 (C) 99 (D) 126 (E) 141

3. You are given the following prices for a stock:

Time	Price
Initial	39
After 1 month	39
After 2 months	37
After 3 months	43

A portfolio of 3-month Asian options, each based on monthly averages of the stock price, consists of the following:

(i) 100 arithmetic average price call options, strike 36.

(ii) 200 geometric average strike call options.

(iii) 300 arithmetic average price put options, strike 41.

Determine the net payoff of the portfolio after 3 months.

(A) 1433 (B) 1449 (C) 1464 (D) 1500 (E) 1512

Questions continue on the next page . . .

4. The price of a 6-month futures contract on widgets is 260.

A 6-month European call option on the futures contract with strike price 256 is priced using Black's formula.

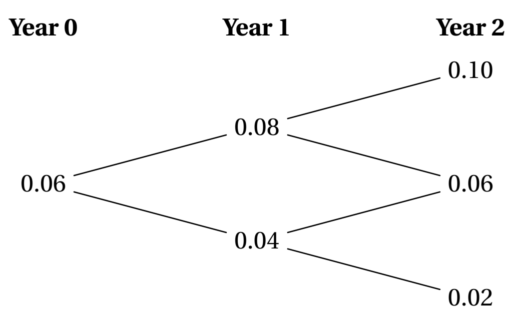
You are given:

(i) The continuously compounded risk-free rate is 0.04.
(ii) The volatility of the futures contract is 0.25.

Determine the price of the option.

(A) 19.84 (B) 20.16 (C) 20.35 (D) 20.57 (E) 20.74

5. You are given the following binomial tree for continuously compounded interest rates:



The probability of an up move is 0.5.

Calculate the continuously compounded interest rate on a default-free 3-year zero-coupon bond.

(A) 0.0593 (B) 0.0594 (C) 0.0596 (D) 0.0597 (E) 0.0598

6. An asset's price at time t, $X(t)$, satisfies the stochastic differential equation

$$dX(t) = 0.5\,dt + 0.9\,dZ(t)$$

You are given that $X(0) = 10$.

Determine $\Pr\big(X(2) > 12\big)$.

(A) 0.16 (B) 0.21 (C) 0.26 (D) 0.29 (E) 0.32

7. For a delta-hedged portfolio, you are given

(i) The stock price is 40.
(ii) The stock's volatility is 0.2.
(iii) The option's gamma is 0.02.

Estimate the annual variance of the portfolio if it is rehedged every half-month.

(A) 0.001 (B) 0.017 (C) 0.027 (D) 0.034 (E) 0.054

Questions continue on the next page ...

8. You own 100 shares of a stock whose current price is 42. You would like to hedge your downside exposure by buying 6-month European put options with a strike price of 40. You are given:

(i) The Black-Scholes framework is assumed.
(ii) The continuously compounded risk-free rate is 5%.
(iii) The stock pays no dividends.
(iv) The stock's volatility is 22%.

Determine the cost of the put options.

(A) 121 (B) 123 (C) 125 (D) 127 (E) 129

9. You are given the following weekly stock prices for six consecutive weeks:

50.02	51.11	50.09	48.25	52.06	54.18

Using the method in the McDonald textbook that does not assume expected return is zero, estimate the annual volatility of the stock.

(A) 0.11 (B) 0.12 (C) 0.29 (D) 0.33 (E) 0.34

10. For European options on a stock having the same expiry and strike price, you are given:

(i) The stock price is 85.
(ii) The strike price is 90.
(iii) The continuously compounded risk free rate is 0.04.
(iv) The continuously compounded dividend rate on the stock is 0.02.
(v) A call option has premium 9.91.
(vi) A put option has premium 12.63.

Determine the time to expiry for the options.

(A) 3 months (B) 6 months (C) 9 months (D) 12 months (E) 15 months

11. You are given the following stochastic differential equations for two geometric Brownian motion processes for the prices of nondividend paying stocks:

$$\mathrm{d}S_1(t) = 0.10S_1(t)\,\mathrm{d}t + 0.06S_1(t)\,\mathrm{d}Z(t)$$
$$\mathrm{d}S_2(t) = 0.15S_2(t)\,\mathrm{d}t + 0.10S_2(t)\,\mathrm{d}Z(t)$$

Determine the continuously compounded risk-free rate.

(A) 0.02 (B) 0.025 (C) 0.03 (D) 0.035 (E) 0.04

12. Which of statements (A)–(D) is *not* a weakness of the lognormal model for stock prices?

(A) Volatility is constant.
(B) Large stock movements do not occur.
(C) Projected stock prices are skewed to the right.
(D) Stock returns are not correlated over time.
(E) (A)–(D) are all weaknesses.

Questions continue on the next page ...

13. For a put option on a stock:

(i) The premium is 2.56.
(ii) Delta is −0.62.
(iii) Gamma is 0.09.
(iv) Theta is −0.02 per day.

Calculate the delta-gamma-theta approximation for the put premium after 3 days if the stock price goes up by 2.

(A) 1.20 (B) 1.32 (C) 1.44 (D) 1.56 (E) 1.62

14. S_t is the price of a stock at time t, with t expressed in years. You are given:

(i) S_t/S_0 is lognormally distributed.
(ii) The continuously compounded expected annual return on the stock is 5%.
(iii) The annual σ for the stock is 30%.
(iv) The stock pays no dividends.

Determine the probability that the stock will have a positive return over a period of three years.

(A) 0.49 (B) 0.51 (C) 0.54 (D) 0.59 (E) 0.61

15. All zero-coupon bonds have a 5% continuously compounded yield to maturity. To demonstrate an arbitrage, you buy one 5-year zero-coupon bond with maturity value 1000 and duration-hedge by buying N 2-year zero-coupon bonds with maturity value 1000. You finance the position by borrowing at the short-term rate.

Determine the amount you borrow.

(A) −1168 (B) −524 (C) 0.389 (D) 524 (E) 1168

16. For an at-the-money European call option on a nondividend paying stock:

(i) The price of the stock follows the Black-Scholes framework
(ii) The option expires at time t.
(iii) The option's delta is 0.5832.

Calculate delta for an-the-money European call option on the stock expiring at time $2t$.

(A) 0.62 (B) 0.66 (C) 0.70 (D) 0.74 (E) 0.82

17. Gap options on a stock have six months to expiry, strike price 50, and trigger 49. You are given:

(i) The stock price is 45.
(ii) The continuously compounded risk free rate is 0.08.
(iii) The continuously compounded dividend rate of the stock is 0.02.

The premium for a gap call option is 1.68.

Determine the premium for a gap put option.

(A) 4.20 (B) 5.17 (C) 6.02 (D) 6.96 (E) 7.95

Questions continue on the next page . . .

18. You are given:

(i) $X(t)$ satisfies the Itô process

$$\frac{dX(t)}{X(t)} = 0.1\,dt + 0.2\,dZ(t)$$

(ii) $X(5) = 40$.

(iii) $Y(t)$ is related to $X(t)$ as follows:

$$Y(t) = 0.1X(t)^2 + 0.9X(t) + 0.1t + 10$$

(iv) $Y(t)$ satisfies

$$dY(t) = \alpha\big(t, Y(t)\big)dt + \sigma\big(t, Y(t)\big)dZ(t)$$

Determine the value of $\alpha\big(t, Y(t)\big)$ for $t = 5$ using Itô's lemma.

(A) 35.70 (B) 35.80 (C) 35.86 (D) 39.85 (E) 42.10

19. A 1-year American pound-denominated put option on euros allows the sale of €100 for £90. It is modeled with a 2-period binomial tree based on forward prices. You are given

(i) The spot exchange rate is £0.8/€.
(ii) The continuously compounded risk-free rate in pounds is 0.06.
(iii) The continuously compounded risk-free rate in euros is 0.04.
(iv) The relative volatility of pounds and euros is 0.1.

Calculate the price of the put option.

(A) 8.92 (B) 9.36 (C) 9.42 (D) 9.70 (E) 10.00

20. For a 1-year call option on a non-dividend paying stock:

(i) The price of the stock follows the Black-Scholes framework.
(ii) The current stock price is 40.
(iii) The strike price is 45.
(iv) The continuously compounded risk-free rate is 0.05.

It has been observed that if the stock price increases 0.50, the price of the option increases 0.25.

Determine the implied volatility of the stock.

(A) 0.32 (B) 0.37 (C) 0.44 (D) 0.50 (E) 0.58

21. The Itô process $X(t)$ satisfies the stochastic differential equation

$$\frac{dX(t)}{X(t)} = 0.1\,dt + 0.2\,dZ(t)$$

Determine $\Pr\big(X(2)^3 > X(0)^3\big)$.

(A) 0.64 (B) 0.66 (C) 0.67 (D) 0.69 (E) 0.71

22. You are simulating one value of a lognormal random variable with parameters $\mu = 1$, $\sigma = 0.4$ by drawing 12 uniform numbers on $[0,1]$. The sum of the uniform numbers is 5.

Determine the generated lognormal random number.

(A) 1.7 (B) 1.8 (C) 1.9 (D) 2.0 (E) 2.1

Questions continue on the next page . . .

23. A nondividend paying stock satisfies the stochastic differential equation

$$\frac{dS(t)}{S(t)} = 0.15\,dt + 0.3\,dZ(t)$$

You are given

(i) The price of the stock is 50.
(ii) The Sharpe ratio is 0.35.

Calculate the price of a European at-the-money call option on the stock with two years to expiry.

(A) 9.62 (B) 10.05 (C) 10.11 (D) 10.29 (E) 10.42

24. You are given:

(i) The price of a stock is 40.
(ii) The continuous dividend rate for the stock is 0.02.
(iii) Stock volatility is 0.3.
(iv) $r = 0.06$.

A 3-month at-the-money European call option on the stock is priced with a 1-period binomial tree. The tree is constructed so that the risk-neutral probability of an up move is 0.5 and the ratio between the prices on the higher and lower nodes is $e^{2\sigma\sqrt{h}}$, where h is the amount of time between nodes in the tree.

Determine the resulting price of the option.

(A) 3.11 (B) 3.16 (C) 3.19 (D) 3.21 (E) 3.28

25. For a portfolio of call options on a stock:

Number of shares of stock	Call premium per share	Delta
100	11.4719	0.6262
100	11.5016	0.6517
200	10.1147	0.9852

Calculate delta for the portfolio.

(A) 0.745 (B) 0.812 (C) 0.934 (D) 297.9 (E) 324.8

Solutions to the above questions begin on page 577.

Appendix A. Solutions for the Practice Exams

Answer Key for Practice Exam 1

1	A	6	B	11	B	16	A	21	E
2	E	7	D	12	C	17	B	22	B
3	B	8	A	13	C	18	E	23	D
4	B	9	D	14	B	19	E	24	B
5	D	10	E	15	A	20	B	25	E

Practice Exam 1

1. [**Section 2.4**] Options are convex, meaning that as the strike price increases, the rate of increase in the put premium does not decrease. The rate of increase from 30 to 35 is $(6.40-2.40)/(35-30)=0.80$, so the rate of increase from 35 to 38 must be at least $(38-35)(0.80)=2.40$, making the price at least $6.40+2.40=8.80$. Thus **(A)** is correct.

2. [**Subsection 1.2.1**] By put-call parity,

$$\begin{aligned}P&=C+Ke^{-rt}-Se^{-\delta t}\\&=4.10+28e^{-0.025}-30=1.4087\end{aligned}$$

For 100 shares, the cost is $100(1.4087)=\boxed{\mathbf{140.87}}$. **(E)**

3. [**Section 13.1**] The monthly arithmetic average of the prices is

$$\frac{39+37+43}{3}=39.6667$$

The monthly geometric average of the prices is

$$\sqrt[3]{(39)(37)(43)}=39.5893$$

The payments on the options are:

- The arithmetic average price call options with strike 36 pay $39.6667-36=3.6667$.
- The geometric average strike call options pay $43-39.5893=3.4107$.
- The arithmetic average price put options with strike 41 pay $41-39.6667=1.3333$.

The total payment on the options is $100(3.6667)+200(3.4107)+300(1.3333)=\boxed{\mathbf{1448.8}}$. **(B)**

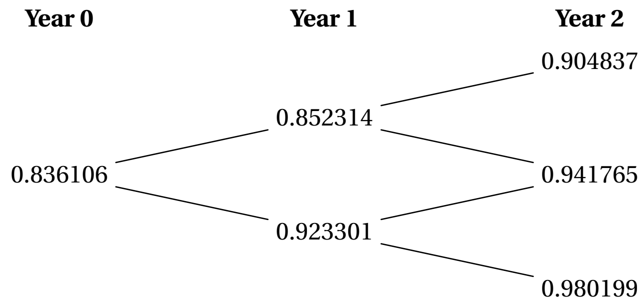


Figure A.1: Zero-coupon bond prices in the solution for question 5

4. [**Section 9.3**] By Black's formula,

$$d_1 = \frac{\ln(260/256) + 0.5(0.25^2)(0.5)}{0.25\sqrt{0.5}} = 0.1761$$
$$d_2 = 0.1761 - 0.25\sqrt{0.5} = -0.0007$$

$$N(d_1) = N(0.18) = 0.5714$$
$$N(d_2) = N(0) = 0.5$$
$$C = 260e^{-0.02}(0.5714) - 256e^{-0.02}(0.5) = \boxed{\mathbf{20.16}} \qquad \textbf{(B)}$$

5. [**Section 18.1**] The prices of bonds at year 2 are $e^{-0.1} = 0.904837$, $e^{-0.06} = 0.941765$, and $e^{-0.02} = 0.980199$ at the 3 nodes. Pulling back, the price of a 2-year bond at the upper node of year 1 is

$$0.5e^{-0.08}(0.904837 + 0.941765) = 0.852314$$

and the price of a 2-year bond at the lower node of year 1 is

$$0.5e^{-0.04}(0.941765 + 0.980199) = 0.923301$$

The price of a 3-year bond initially is

$$0.5e^{-0.06}(0.852314 + 0.923301) = 0.836106$$

The yield is $-(\ln 0.836106)/3 = \boxed{\mathbf{0.059667}}$. **(D)** The binomial tree of bond prices is shown in figure A.1.

6. [**Lesson 16**] This is arithmetic Brownian motion with $\mu = 0.5$, $\sigma = 0.9$. For time $t = 2$, $\mu t = (0.5)(2) = 1$, $\sigma\sqrt{t} = 0.9\sqrt{2}$. The movement is $12 - 10 = 2$.

$$\Pr(X(2) > 12) = 1 - N\left(\frac{2-1}{0.9\sqrt{2}}\right) = 1 - N(0.79) = 1 - 0.7852 = \boxed{\mathbf{0.2148}} \qquad \textbf{(B)}$$

7. [**Section 12.4**] By the Boyle-Emanuel formula, with period $\frac{1}{24}$ of a year, the variance of annual returns is

$$\mathrm{Var}(R_{1/24,1}) = \tfrac{1}{2}\Big((40^2)(0.20^2)(0.02)\Big)^2/24 = \boxed{\mathbf{0.0341}} \qquad \textbf{(D)}$$

8. [**Lesson 9**] For one share, Black-Scholes formula gives:

$$d_1 = \frac{\ln(42/40) + [0.05 - 0 + 0.5(0.22^2)](0.5)}{0.22\sqrt{0.5}} = 0.5521$$
$$d_2 = 0.5521 - 0.22\sqrt{0.5} = 0.3966$$

$$N(-d_2) = N(-0.40) = 0.3446$$
$$N(-d_1) = N(-0.55) = 0.2912$$
$$P = 40e^{-0.05(0.5)}(0.3446) - 42(0.2912) = 1.2133$$

The cost of 100 puts is $100(1.2133) = \boxed{\mathbf{121.33}}$. **(A)**

Note that this question has nothing to do with delta hedging. The purchaser is merely interested in guaranteeing that he receives at least 40 for each share, and does not wish to give up upside potential. A delta hedger gives up upside potential in return for keeping loss close to zero.

9. [**Lesson 8**] First calculate the logarithms of ratios of consecutive prices

t	S_t	$\ln(S_t/S_{t-1})$
0	50.02	
1	51.11	0.02156
2	50.09	−0.02016
3	48.25	−0.03743
4	52.06	0.07600
5	54.18	0.03991

Then calculate the sample standard deviation.

$$\frac{0.02156 - 0.02016 - 0.03743 + 0.07600 + 0.03991}{5} = 0.01598$$
$$\frac{0.02156^2 + 0.02016^2 + 0.03743^2 + 0.07600^2 + 0.03991^2}{5} = 0.001928$$
$$\tfrac{5}{4}(0.001928 - 0.01598^2) = 0.002091$$
$$\sqrt{0.002091} = 0.04573$$

Then annualize by multiplying by $\sqrt{52}$

$$0.04573\sqrt{52} = \boxed{\mathbf{0.3298}} \qquad \textbf{(D)}$$

10. [**Subsection 1.2.1**] By put-call parity

$$12.63 - 9.91 = 90e^{-0.04t} - 85e^{-0.02t}$$
$$90e^{-0.04t} - 85e^{-0.02t} - 2.72 = 0$$

Let $x = e^{-0.02t}$ and solve the quadratic for x.

$$x = \frac{85 + \sqrt{85^2 + 4(90)(2.72)}}{2(90)} = \frac{175.577}{180} = 0.975428$$

The other solution to the quadratic leads to $x<0$, which is impossible for $x=e^{-0.02t}$. Now we solve for t.

$$\begin{aligned}
e^{-0.02t} &= 0.975428\\
0.02t &= -\ln 0.975428 = 0.024879\\
t &= 50(0.024879) = \boxed{\mathbf{1.244}} \qquad \textbf{(E)}
\end{aligned}$$

11. [**Section 17.3**] The Sharpe ratios must be equal, so

$$\begin{aligned}
\frac{0.10-r}{0.06} &= \frac{0.15-r}{0.10}\\
0.01-0.1r &= 0.009-0.06r\\
0.04r &= 0.001\\
r &= \boxed{\mathbf{0.025}} \qquad \textbf{(B)}
\end{aligned}$$

12. [**Subsection 6.2.1**] **(C)** is not a weakness, since one would expect that the multiplicative change in stock price, rather than the additive change, is symmetric.

13. [**Section 12.2**] Theta is expressed per day of decrease, so we just have to multiply it as given by 3. Thus the change in price is

$$\Delta\epsilon + 0.5\Gamma\epsilon^2 + \theta h = -0.62(2) + 0.5(0.09)(2^2) - 0.02(3) = -1.12$$

The new price is $2.56 - 1.12 = \boxed{\mathbf{1.44}}$. **(C)**

14. [**Section 7.2**] We are given that the average return $\alpha = 0.05$, so the parameter of the associated normal distribution is $\mu = 0.05 - 0.5(0.3^2) = 0.005$. For a three year period, $m = \mu t = 0.015$ and $v = \sigma\sqrt{t} = 0.3\sqrt{3} = 0.5196$. For a positive return, we need the normal variable with these parameters to be greater than 0. The probability that an $\mathcal{N}(0.015, 0.5196^2)$ variable is greater than 0 is $N(0.015/0.5196) = N(0.03) = \boxed{\mathbf{0.512}}$. **(B)**

15. [**Subsection 20.1**] The price of a 5-year bond with maturity value 1 is $e^{-5(0.05)} = 0.778801$. The price of a 2-year bond with maturity value 1 is $e^{-2(0.05)} = 0.904837$. Letting $P(0,T)$ be the price of a T-year zero-coupon bond, the duration-hedge ratio is

$$N = -\frac{5P(0,5)}{2P(0,2)} = -\frac{5(0.778801)}{2(0.904837)} = -2.15177$$

Thus you buy a 5-year bond for 1000 and sell 2.15177 2-year bonds for 1000 at a cost of

$$778.80 - 2.15177(904.83) = \boxed{\mathbf{-1168}} \qquad \textbf{(A)}$$

16. [**Section 10.1**] Delta is $e^{-\delta t}N(d_1)$, or $N(d_1)$ for a non-dividend paying stock. Since the option is at-the-money,

$$d_1 = \frac{(r+0.5\sigma^2)t}{\sigma\sqrt{t}} = \frac{r+0.5\sigma^2}{\sigma}\sqrt{t}$$

So doubling time multiplies d_1 by $\sqrt{2}$.

$$N(d_1) = 0.5832$$

$$\begin{aligned} d_1 &= N^{-1}(0.5832) = 0.21 \\ d_1\sqrt{2} &= (0.21)(1.4142) = 0.2970 \\ N(0.2970) &= N(0.30) = 0.6179 \qquad \textbf{(A)} \end{aligned}$$

17. [**Section 14.2**] For gap options, put-call parity applies with the strike price. If you buy a call and sell a put, if $S_T > K_2$ (the trigger price) you collect S_T and pay K_1, and if $S_T < K_2$ you pay K_1 and collect S_T which is the same as collecting S_T and paying K_1, so

$$C - P = Se^{-\delta t} - K_1 e^{-rt}$$

In this problem,

$$P = C + K_1 e^{-rt} - Se^{-\delta t} = 1.68 + 50e^{-0.04} - 45e^{-0.01} = \boxed{\mathbf{5.167}} \qquad \textbf{(B)}$$

18. [**Section 17.1**] By Itô's lemma,

$$\begin{aligned} \mathrm{d}Y(t) &= Y_X \mathrm{d}X + \tfrac{1}{2} Y_{XX} (\mathrm{d}X)^2 + Y_t \mathrm{d}t \\ &= \Big(0.1(2)X(t) + 0.9\Big)\mathrm{d}X + \tfrac{1}{2}(2)(0.1)(\mathrm{d}X)^2 + 0.1\,\mathrm{d}t \\ &= \Big(0.2X(t) + 0.9\Big)\Big(0.1X(t)\mathrm{d}t + 0.2X(t)\mathrm{d}Z\Big) + 0.1\Big(0.2^2 X(t)^2 \mathrm{d}t\Big) + 0.1\,\mathrm{d}t \end{aligned}$$

We only need the $\mathrm{d}t$ term at $t = 5$, which is

$$\begin{aligned} &\Big(0.2X(5) + 0.9\Big)\Big(0.1X(5)\Big) + 0.004X(5)^2 + 0.1 \\ &= (8.9)(4) + (0.004)(40^2) + 0.1 \\ &= \boxed{\mathbf{42.10}} \qquad \textbf{(E)} \end{aligned}$$

19. [**Section 4.3**] The 6-month forward rate of euros in pounds is $e^{(0.06-0.04)(0.5)} = e^{0.01} = 1.01005$. Up and down movements, and the risk-neutral probability of an up movement, are

$$\begin{aligned} u &= e^{0.01+0.1\sqrt{0.5}} = 1.08406 \\ d &= e^{0.01-0.1\sqrt{0.5}} = 0.94110 \\ p^* &= \frac{1.01005 - 0.94110}{1.08406 - 0.94110} = 0.4823 \\ 1 - p^* &= 1 - 0.4823 = 0.5177 \end{aligned}$$

The binomial tree is shown in figure A.2. At the upper node of the second column, the put value is calculated as

$$P_u = e^{-0.03}(0.5177)(0.08384) = 0.04212$$

At the lower node of the second column, the put value is calculated as

$$P_d^{\text{tentative}} = e^{-0.03}\Big((0.4823)(0.08384) + (0.5177)(0.19147)\Big) = 0.13543$$

but the exercise value $0.9 - 0.75288 = 0.14712$ is higher so it is optimal to exercise. At the initial node, the calculated value of the option is

$$P^{\text{tentative}} = e^{-0.03}\Big((0.4823)(0.04212) + (0.5177)(0.14712)\Big) = 0.09363$$

Since $0.9 - 0.8 = 0.1 > 0.09363$, it is optimal to exercise the option immediately, so its value is 0.10 (which means that such an option would never exist), and the price of an option for €100 is $100(0.10) = \boxed{\mathbf{10}}$. **(E)**

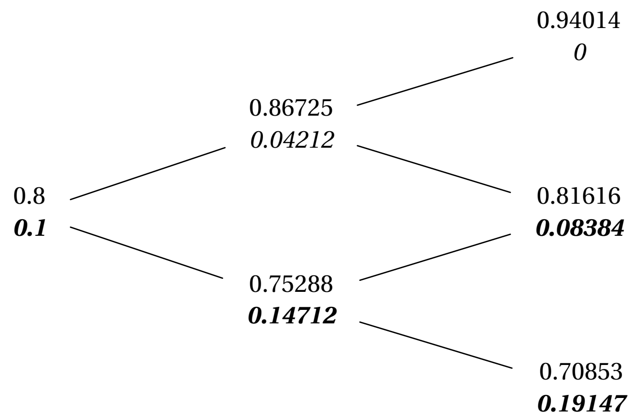


Figure A.2: Exchange rates and option values for put option of question 19

20. [**Section 11.2.1**] Δ is observed to be $0.25/0.50 = 0.5$. In Black-Scholes formula, $\Delta = e^{-\delta t}N(d_1) = N(d_1)$ in our case. Since $N(d_1) = 0.5$, $d_1 = 0$. Then

$$\frac{\ln(S/K) + r + 0.5\sigma^2}{\sigma} = 0$$
$$\ln(40/45) + 0.05 + 0.5\sigma^2 = 0$$
$$0.5\sigma^2 = -\ln(40/45) - 0.05 = 0.11778 - 0.05 = 0.06778$$
$$\sigma^2 = \frac{0.06778}{0.5} = 0.13556$$
$$\sigma = \sqrt{0.13556} = \boxed{\mathbf{0.3682}} \qquad \textbf{(B)}$$

21. [**Section 7.2**] The fraction $X(2)/X(0)$ follows a lognormal distribution with parameters $m = 2\bigl(0.1 - 0.5(0.2^2)\bigr) = 0.16$ and $v = 0.2\sqrt{2}$. Cubing does not affect inequalities, so the requested probability is the same as $\Pr\bigl(\ln X(2) - \ln X(0) > 0\bigr)$, which is

$$1 - N\left(\frac{-0.16}{0.2\sqrt{2}}\right) = N(0.57) = \boxed{\mathbf{0.7157}} \qquad \textbf{(E)}$$

22. [**Section 15.2**] The sum of the uniform numbers has mean 6, variance 1, so we subtract 6 to standardize it.

$$5 - 6 = -1$$

We then multiply by σ and add μ to obtain a $\mathcal{N}(\mu, \sigma^2)$ random variable.

$$(-1)(0.4) + 1 = 0.6$$

Then we exponentiate.

$$e^{0.6} = \boxed{\mathbf{1.822}} \qquad \textbf{(B)}$$

23. [**Section 17.3**] We back out the risk-free rate from the Sharpe ratio, as defined in equation (17.3):

$$0.35 = \frac{0.15 - r}{0.3}$$
$$r = 0.15 - 0.3(0.35) = 0.045$$

The given Itô process is a geometric Brownian motion. A stock following a geometric Brownian motion is in the Black-Scholes framework, so we use the Black-Scholes formula for the price of the option. Note that $K = S = 50$ since the option is at-the-money.

$$d_1 = \frac{\big(0.045 + 0.5(0.3^2)\big)(2)}{0.3\sqrt{2}} = 0.4243$$
$$d_2 = 0.4243 - 0.3\sqrt{2} = 0$$
$$N(d_1) = N(0.42) = 0.6628$$
$$N(d_2) = N(0) = 0.5$$
$$C(50, 50, 2) = 50(0.6628) - 50e^{-0.045(2)}(0.5) = \boxed{\mathbf{10.29}} \qquad \textbf{(D)}$$

24. [**Lesson 3**] The risk-neutral probability is

$$0.5 = p^* = \frac{e^{(r-\delta)t} - d}{u - d} = \frac{e^{(0.06-0.02)(0.25)} - d}{u - d} = \frac{e^{0.01} - d}{u - d}$$

but $u = d e^{2\sigma\sqrt{h}} = d e^{2(0.3)(1/2)} = d e^{0.3}$, so

$$e^{0.01} - d = 0.5\left(e^{0.3}d - d\right)$$
$$e^{0.01} = d\left(0.5(e^{0.3} - 1) + 1\right) = 1.17493d$$
$$d = \frac{e^{0.01}}{1.17493} = 0.85967$$
$$u = 0.85967e^{0.3} = 1.16043$$

The option only pays at the upper node. The price of the option is

$$C = e^{-rh}p^*(Su - K) = e^{-0.06(0.25)}(0.5)\big(40(1.16043) - 40\big) = \boxed{\mathbf{3.1609}} \qquad \textbf{(B)}$$

25. [**Subsection 10.1.7**] Delta for a portfolio of options on a single stock is the sum of the individual deltas of the options.

$$100(0.6262) + 100(0.6517) + 200(0.9852) = \boxed{\mathbf{324.8}} \qquad \textbf{(E)}$$